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Failure in laminates in tension under increasing stress, constant stress, and cyclic stress

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Both monotonic and cyclic tension experiments have been carried out to fracture on transparent laminates made from flat ribbon glass and polyethylene sheets by heat bonding in a vacuum. The distribution of the measured tensile fracture stresses in monotonic loading correlates very well with the distribution of fracture stresses calculated from the measured distribution of element fracture stresses and the yield strength in shear of the polyethylene, according to a detailed statistical theory worked out earlier by Scop and Argon. Although the expected mode of fatigue damage by propagation of delamination cracks emanating from isolated stable fractures in reinforcing elements was observed, actual fatigue failure was a result of a more rapid mechanism of continued fracturing in reinforcing elements by a humidity-induced time-integrated static fatigue process. While laminates subjected to static stresses of the same magnitude as those in the dynamic experiments failed by the development of an identical form of damage during the same length of time under stress in laboratory air, other laminates tested in the same manner in dry air had 5 to 10 fold increased lives.

In addition re-testing of individual elements of delaminated composites showed that elements can often be damaged during lamination, which must be taken into account in any quantitative study.

1. Introduction

The tensile strength of composites made of strong, brittle, and continuous elements bonded by an elastically compliant or plastically deformable matrix has been studied extensively. For recent reviews see Rosen and Dow [1], and Argon [2, 3]. The fracture in tension of initially unnotched composites for known variability of the strength of individual elements, and elastic-plastic properties of the matrix, is now well understood both qualitatively and quantitatively [1-6]. The degradation of such composites under repeated tensile loading in a fatigue mode is, however, not equally well understood.

It has been shown earlier [2] that if the surface flaw distribution function in the brittle reinforcement layers of a laminate* can be described by a simple power function as was originally done by Weibull [7] then the ratio of the average strength of the laminate to the average strength of the isolated element (for a given laboratory test length) is greater than unity for flaw distribution function exponents $m < 10$. When such laminates with elements having exponents m of the flaw distribution function less than 10 are subjected to tensile loading, isolated fractures can occur in the elements from bad flaws at stresses less than the laminate strength. Because of the relatively high variability of the strength of elements, however, the stress concentrations produced by such early fractures can be tolerated by the surrounding elements which have a high probability of being strong in these areas of local overstressing. Hence, many isolated element fractures are observable in such laminates prior to the final fracture of the laminate as a whole [8]. Alternatively, when the exponent of the flaw distribution function m of the elements

*Here we use the word laminate to describe a layer-like composite made of alternating planar layers of strong and brittle reinforcement bonded together with a ductile matrix.

exceeds about 10, the elements possess a high average strength with low variability, so that when, finally, an individual element fails from a bad flaw, it has a high probability of overstressing the surrounding elements at the site of the initial break leading to progressive fracture of adjacent elements and final fracture of the whole laminate. In such cases, the average strength of the laminate is less than the average strength of the elements, and very few, if any, stable element fractures are observable in the laminate prior to total fracture of the whole laminate. In nearly all cases of reinforcement using glass, boron, or carbon fibre (unless extreme precautions are taken), the former case holds, the exponent is less than 10, and the laminate is stronger than the elements. It must be emphasized here that the above discussion which is against the widely accepted "rule of mixtures" results directly from the statistics of interacting reinforcing elements, coupled by a traction transmitting matrix [6, 2, 9].

When a laminate of this regular type is subjected to repeated tension two limiting modes of fatigue can be conceived. First, if the yield strain of the matrix is less than the average fracture strain of the reinforcing elements, repeated tensile loading can produce fatigue of the matrix material without any significant fracturing of the reinforcing elements. The laminate would then delaminate, lose its bending stiffness, and would become generally useless without, however, actually undergoing fracture [10]. Alternatively, the yield strain of the matrix could be higher than the average fracture strain of the elements. Then cycling below the tensile strength of the laminate could produce isolated stable fractures in the reinforcing elements where, locally, the yield strength of the matrix could be exceeded and the matrix could be gradually fatigued by cyclic sliding of the broken element ends. This could then produce propagation of delamination zones starting from the initial element breaks. When such delamination zones bridge a number of isolated fractures at different levels in adjoining elements, an effective crack of critical length could then be produced which would fracture the laminate in one final quarter cycle. If the reinforcing elements are susceptible to static fatigue or stress corrosion cracking, additional time-dependent effects could also be present.

Here we report the results of a study on an idealized laminate composed of glass ribbon and polyethylene sheets subjected to tensile loads that are either constant, monotonically increasing, or repeatedly applied.

2. Experimental procedures

In order to observe internal fracture processes it was decided to make the laminates transparent by constructing them from alternating sheets of glass ribbon as reinforcement and polyethylene sheets as ductile matrix. The glass ribbon used in the laminates is manufactured by the Corning Co as a dielectric material for capacitors. It is made of a potash-lead glass with a nominal composition of 49% PbO, 42% SiO_2, 6% KO, 2% NaO and 1% LiO (their designation no. 8871). Since the ribbon is made by a continuous process of drawing out from a melt, it is thin, flat, of rather uniform dimensions, has fire-polished surfaces all over, and has little variation of physical and mechanical properties along a spool of a thousand foot length. All experiments to be reported here were made with ribbon having cross-sectional dimensions of $2.39 \times 10^{-3} \times 0.463$ in. coming from the same spool. The polyethylene sheets used for lamination were of a thickness of about 5×10^{-4} in.

Five-layer laminates were made by using the following procedure. Lengths of glass ribbon 6 in. long were cut from the spool and washed in a warm water solution of a mild detergent for cleaning laboratory glassware. They were rinsed in alcohol and dried in a stream of hot air. The polyethylene sheets were similarly cleaned and dried. A five-layer stack was then prepared as shown in Fig. 1 by threading a continuous sheet of polyethylene four times through slots in a metal frame (not shown in the sketch). Five previously prepared lengths of glass ribbon were then put between the four folds of the polyethylene sheets, and were carefully aligned to lie on top of each other. A rectangular metal block was put on top of the stack to prevent the glass sheets from shifting and to provide a pressurizing force in the final fusion process. The upper and lower glass sheets were separated from the metal block and the base of the assembly frame by thin teflon sheets. This prevented sticking of the laminates to the block or frame and minimized external mechanical damage. The whole assembly was then put into a vacuum oven where the glass ribbons and polyethylene sheets were then fused together by raising the temperature to 225°C after several hours of pump-out.

Laminates prepared in this manner were

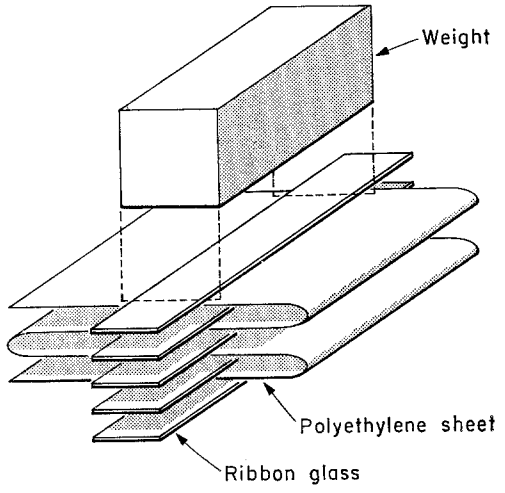

Figure 1 Schematic representation of lamination procedure.

perfectly transparent, free of any air pockets and could not be peeled apart without fracturing the glass sheets. The total thickness of the laminates was then about 0.014 in. giving, for the laminate, a volume fraction V_r of reinforcement of 0.86. Two 2 in. long strips of low carbon steel of 0.02 × 0.5 in. cross-sectional dimensions were then glued with Eastman 910 cement on each side of both ends of the laminate to serve as grips, leaving a 3.0 in. gauge length in the middle. Although this method of fastening did not completely eliminate occurrence of fracture near the grips, the fracture stresses of specimens where failure near grips did occur had the same mean value and spread as those where failure occurred in the gauge section. In addition to the various monotonic loading and repeated loading experiments on such five-layer laminates, many tensile fracture experiments were performed on individual ribbon sheets to determine their flaw distribution function. Here, too, loading was through a pair of steel strips glued to both ends of the sheet. In such experiments, gauge lengths of both 0.75 and 3.0 in. were used.

Monotonic tension testing was done in an Instron machine at conventional extension rates (10^{-3} in. min^{-1}). Repeated loading experiments were performed on a Baldwin SF-1U tensile fatigue machine with automatic mean load control, and operating synchronously at a cyclic frequency of 30 Hz. Static fatigue experiments were performed by dead weight loading in an incubator with controlled atmospheres. Many of the monotonically and cyclically loaded laminates were observed through a microscope to study the development of damage.

3. Experimental results

3.1. Strength of constituents

To determine the distribution function of the strength-limiting flaws on the glass ribbons, one hundred single element specimens with a gauge length of 0.75 in. and having glued steel strips for gripping were prepared as described above, by taking ribbon material directly from the spool without the washing treatment used for making laminates. These specimens were then tested to fracture in an Instron machine and their fracture stresses were calculated. The resulting fracture stresses are shown in Fig. 2 in ascending order to

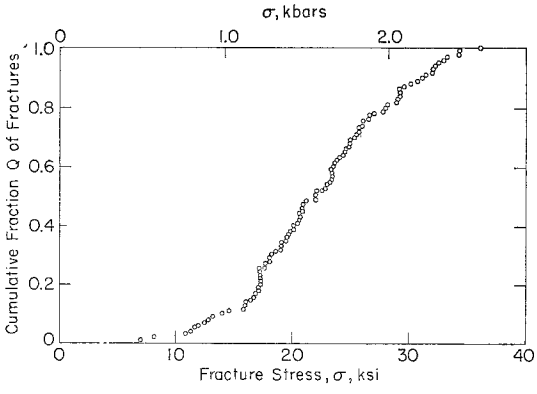

Figure 2 Cumulative distribution of fracture stresses in unwashed elements for elements of 0.75 in. gauge length.

represent the cumulative probability $Q(\sigma)$ of fracture at a stress σ or less, of elements of the given dimensions. If the expected cumulative distribution function of flaws $\xi(\sigma)$ per unit area is a simple power function of the type,

$$\xi(\sigma) = C\sigma^m \tag{1}$$

as used by Weibull [7] where C has the dimensions of stress $^{-m}$ area^{-1} then the cumulative probability of fracture of a specimen of length L and width w at a stress of σ or less is the well-known expression [7]

$$Q(\sigma) = 1 - \exp(-s) \tag{2}$$

where s is a non-dimensional stress

$$s = LwC\sigma^m. \tag{3}$$

The exponent, m, and the coefficient, C, of the flaw distribution function can be readily

determined from a double logarithmic plot of $\{-\ln[1 - Q(\sigma)]\}$ against σ, where the slope of the straight line immediately gives m and evaluation of the function with this exponent, at any point, gives C. That the flaw distribution function is of this expected form is shown in Fig. 3. From this plot it is readily found that the exponent $m = 4.2$ and the coefficient $C = 5.92 \times 10^{-6}$ in.$^{-2}$ ksi$^{-4.2}$*.

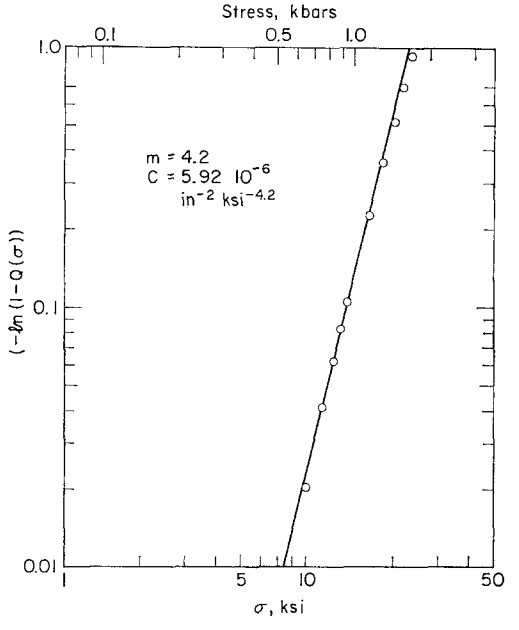

Figure 3 Logarithmic plot showing that the flaw distribution function on the elements is of a simple power function type.

Some lap-shear tests were performed on microscope slides stuck together with polyethylene. These, however did not yield reliable results. For this reason the yield stress in shear of polyethylene was measured on bulk torsion specimens [11] and was found to be $k = 2.12$ ksi. Examination of fracture surfaces of laminates furnished ample evidence that polyethylene as used here is a superior matrix material which is not only elastically compliant but also remarkably ductile (see Figs. 7 and 8). We note, in passing, that the shear modulus of polyethylene at room temperature is 28.0 ksi while the Young's modulus for the lead-potash glass is 8.4 ksi. This makes the tensile yield strain of the polyethylene, $\epsilon_y = 0.037$ and the average tensile fracture strain of the glass ribbon (for an average fracture stress of 21 ksi from Fig. 2) only $\epsilon_f = 0.0025$.

3.2. Lamination damage

There has been occasional evidence that the process of lamination damages reinforcing elements and alters their flaw distribution function. To check the possibility of such damage in lamination, a set of experiments was performed in which a number of manufactured laminates were delaminated by soaking them in hot xylene to dissolve away the polyethylene matrix followed by re-testing the delaminated glass ribbon elements to determine if any change in their strength distribution had occurred. Since a number of new steps are introduced in this procedure, however, it was equally important to determine first the possible effect of these steps on the mechanical properties of the reinforcing elements. Since these new element tests were performed on specimens having a 3 in. gauge length it was first necessary to compute the new strength distribution for this new gauge length from the initial distribution presented in Fig. 2 for specimens with a 0.75 in. gauge length, by using Equations 2 and 3. The new distribution curve obtained in this manner is shown in Figs. 4 and 5. In Fig. 4, the effect of washing the specimens with the laboratory detergent is shown as a distinct rise in the fracture stress of the individual elements of ribbon material. Since this effect was unexpectedly strong, the glass ribbon coming directly from the spool was examined in a microscope. This showed that it was covered with a thin film of an oily substance which had also formed an extensive network of microscopic "crystallite flowers" on the surface of the glass. All this surface deposit, which could be readily washed off with the laboratory detergent solution, was traced to come from a slightly waxy strip of paper wrapped between the layers of glass ribbon to prevent them from touching each other. The nature of this waxy substance, which appeared to have a strong stress corrosion property, could not be identified. Fig. 4 also shows the effect of immersion of unwashed glass ribbon in hot xylene. It appears that apart from some decreased variability in strength relative to the washing treatment, immersion in hot xylene produces the same effect on strength as washing. Finally, the glass ribbon segments obtained by actual delamination in hot xylene of previously manufactured laminates were tested to give a

*1 ksi = 10^3 psi = 6.89 N mm^{-2}.

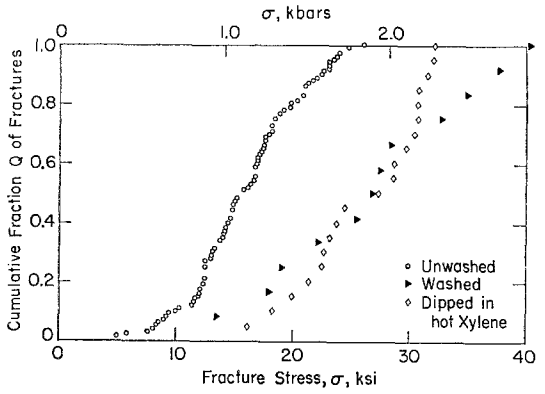

Figure 4 Cumulative distribution of fracture stresses in unwashed, washed specimens, and in unwashed specimens dipped in hot xylene. All 3.0 in. gauge lengths.

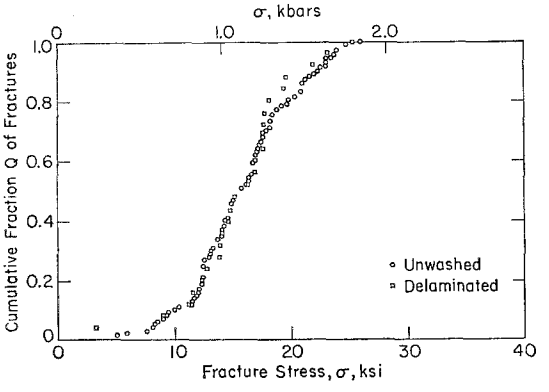

Figure 5 Cumulative distribution of fracture stresses in unwashed elements, and elements re-tested after delamination of laminates with hot xylene; all 3.0 in. gauge lengths.

strength distribution shown in Fig. 5 which falls most remarkably on the initial distribution of strengths of the *unwashed* specimens.

Since the higher strength resulting from the washing was not a transitory effect and no trivial explanation could be found for it, it was concluded that the distribution of strengths given by the black triangles represents the inherent strength of the glass ribbon and that the stress corrosion effect of the oily substance has the same strength reduction effect as damage resulting from lamination. Although this interpretation is preferred, it cannot be ruled out that the increased strength owing to washing and immersion in hot xylene is an artifact and that lamination produces no damage at all. For the purposes of this study, however, it is sufficient to know that, whatever the reason, the actual strength distribution of the elements in the laminate are given accurately by the statistical parameters in Fig. 3.

3.3. Tensile fracture of laminates

Some samples of the five-layer laminates with 3 in. gauge length were tested in tension to determine a tensile reference strength prior to any fatigue experiments. The measured fracture stresses, σ_c, for the laminates are arranged in ascending order in Fig. 6 to give a cumulative

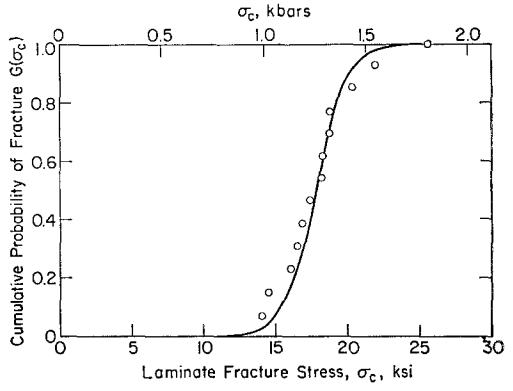

Figure 6 Cumulative distribution of laminate fracture strengths (open circles), plotted together with the distribution curve computed from the statistical fracture theory of Scop and Argon [4].

probability $G(\sigma)$ for laminate fracture with an average tensile fracture stress of 18.3 ksi. The theoretical distribution of tensile strengths of the laminates based on the measured flaw distribution function on the reinforcing elements and the yield strength in shear of the polyethylene matrix, given earlier, was calculated from the detailed statistical fracture theory of Scop and Argon [4] for laminates *with few numbers of elements* (see Appendix 1 for details). The resulting theoretical curve, which has no adjustable parameters, is also shown in Fig. 6. The agreement between theory and experiment is considered very good.

The fracture surfaces of the broken laminates were studied under the microscope. Ample evidence was found that the polyethylene matrix performed in a plastic, and ductile manner. Fig. 7 shows characteristic ductile fracture dimples on a polyethylene layer fractured in shear by relative sliding of two sheets of glass ribbon, while Fig. 8 shows a piece of polyethylene sheet drawn out of the gap between two glass sheets at the fracture surface.

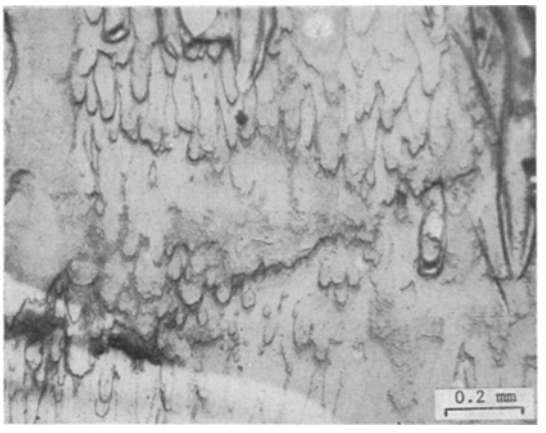

Figure 7 Ductile fracture dimples on a shear fracture surface of polyethylene between two glass ribbon sheets.

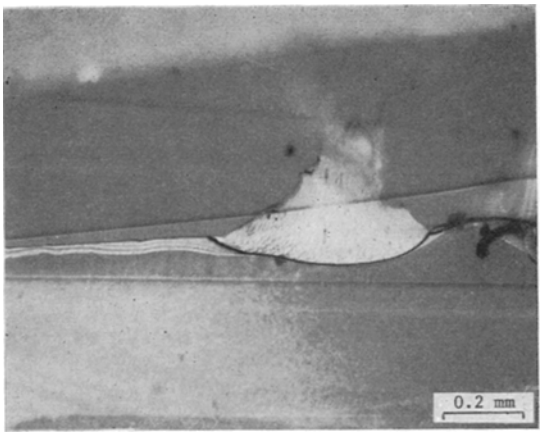

Figure 8 Plastically drawn polyethylene matrix sheet near fracture surface.

3.4. Tension-fatigue experiments

As discussed in Section 3.1 the tensile yield strain of the polyethylene matrix is 15 times as large as the tensile fracture strain of the reinforcing ribbon. Hence, it was expected that repeated application of a tensile stress at a level less than the composite strength, σ_c, would produce fatigue damage by the second of the two limiting mechanisms discussed in the introduction. That is, delamination cracks should form from the stable isolated cracks in the elements that are expected to develop during the first maximum excursion of the stress at a level below σ_c in this laminate in which the exponent m in the element flaw distribution function is 4.2. Such delamination cracks should then propagate parallel to the reinforcing sheets to bridge some of the isolated primary cracks in elements until a long enough effective crack is formed to result in rapid fracture in the last quarter cycle. The fatigue S-N curve of the laminates is shown in Fig. 9 in relation to the tensile strength of the laminates. As the figure shows, the S-N curve is extremely flat and a small decrease in the applied stress amplitude produces a large change in fatigue life. Fig. 10 shows the development of damage in a specimen subjected to a repeated application of a tensile stress of 9.75 ksi. Two isolated cracks in elements are seen to have formed during the application of the mean load. The expected propagation of delamination cracks can be seen in Fig. 10 after 3000 cycles in connection with the two fractures formed during the pre-load. The figure, however, also shows the formation of additional fractures in elements with increasing number of cycles. In particular, the picture after 7000 cycles shows that a massive fracture has appeared in an element next to one having the thin crack in the middle of the gauge section at 3000 cycles. The last picture shows that final fracture of the laminate has occurred after 8000 cycles in this area of earlier massive fracture. Because this continued appearance of new cracks under a condition where both the glass reinforcement and the polyethylene matrix was cycling elastically was totally unexpected, a study was made of the development of such cracking in a number of specimens. Fig. 11 shows, for three specimens, how such continued fracturing occurred under a constant amplitude of repeatedly applied stress. Examination of partially cycled but unfractured specimens under the

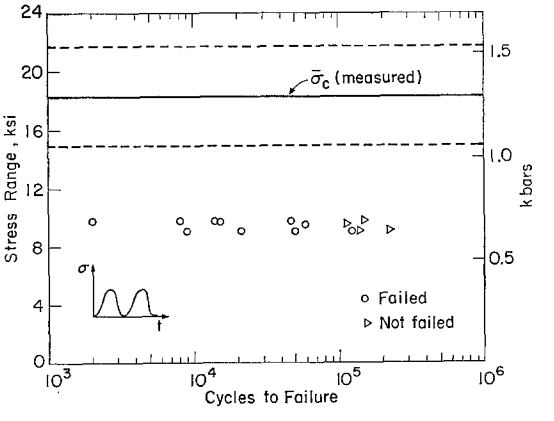

Figure 9 Fatigue S-N curve for five-layer laminate stressed in a tension-release mode.

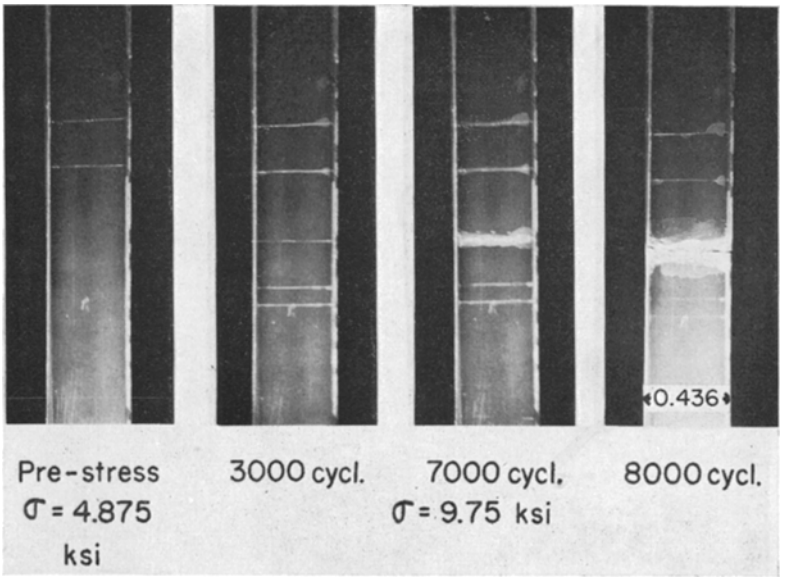

Figure 10 Development of damage by continued fracturing of glass-ribbon elements in a cycling laminate.

microscope revealed no obvious clue. Since glass is normally susceptible to static fatigue by stress corrosion cracking in a humid environment, the possibility of integrated static fatigue under load cycling was suspected, and investigated as discussed in the following section.

For the laminates of the given size, subjected to a peak excursion stress of 9.05 ksi, the expectation value of the number of stable fractures in elements is obtainable from Equation 3 by multiplying it with the number of elements (5). This would give only about one crack which agrees with the number observed on first application of the stress. The expectation value of stable cracks in the same laminate fractured by monotonically loading to an average stress of 18.3 ksi, on the other hand, is nearly 10. We see from Fig. 11 that when the fatigue specimens finally fracture, the number of stable cracks they contain are close to this value for monotonic fracturing under an increasing tensile stress, suggesting further that a continued degradation process is at work under seemingly elastic extension and release.

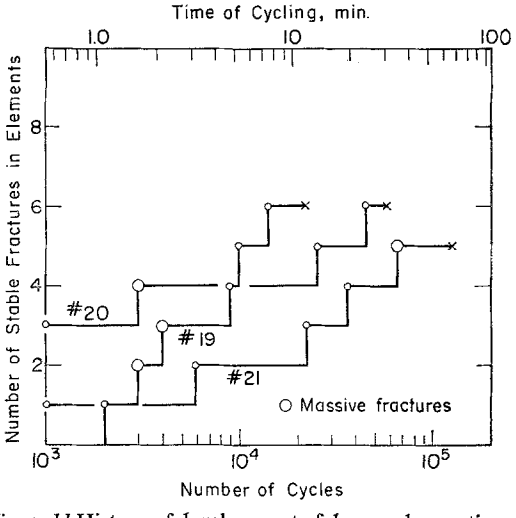

Figure 11 History of development of damage by continued fracture in elements in three laminates in tension-release cycling, for 3.0 in. test length ($\sigma_{19} = 9.05$ ksi, $\sigma_{20} = 9.58$ ksi, $\sigma_{21} = 9.05$ ksi).

3.5. Static fatigue in individual elements and in laminates

The susceptibility of the glass ribbon material to static fatigue was investigated by testing a number of unwashed, individual element specimens of 3 in. gauge length at a conventional rate of loading in an Instron machine with an isolation chamber containing a laboratory desiccant. The fracture stresses, arranged in ascending order, are shown in Fig. 12 in relation to the control distribution of the unwashed specimens. Clearly, the absence of humidity produces a very significant rise in average fracture strength indicating that static fatigue was indeed a possibility.

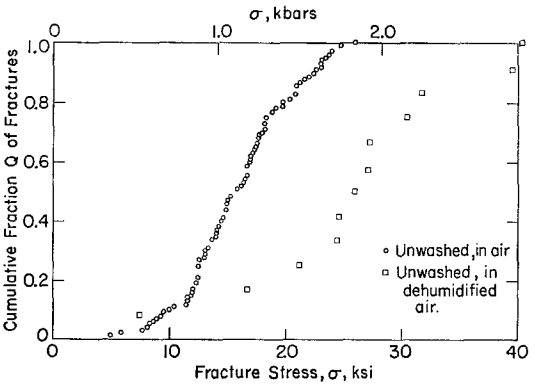

Figure 12 Cumulative distribution of fracture strengths in unwashed elements tested in normal humid air, and unwashed elements tested in dry air, all having a 3.0 in. gauge length.

Since most of the surfaces of the reinforcing elements in a laminate are internal surfaces, protected by polyethylene, static fatigue tests were performed on a number of laminates by subjecting them to a constant stress of 9.05 ksi in a normal laboratory atmosphere. Some of these laminates were carefully observed while under constant load to record time-dependent fracturing of elements. Fig. 13 shows the history of seven such laminates under a constant stress in laboratory air. Cracks in elements continue to appear in time until final fracture occurs when the total number of fractured elements is between three and four. When the average time under load in these statically loaded experiments is compared with the integrated time under load in the repeated tensioning experiments shown in Fig. 11, it is seen that they are of the same magnitude.

To see whether elimination of humidity increases the life of laminates under a constant stress, seven additional laminates were subjected to the same constant stress of 9 05 ksi in an incubator filled with dry air. The results shown in Fig. 13 indicate that in four out of seven cases there has been an increase in the time to fracture over the longest time to fracture in normal laboratory air. There is also an increase in the average number of stable fractures in elements prior to the final fracture of the laminate, which remains unexplained. Again, however, the number of actual stable fractures counted prior to final fracture is nearer the expectation value for fractures in elements in a laminate monotonically loaded to fracture.

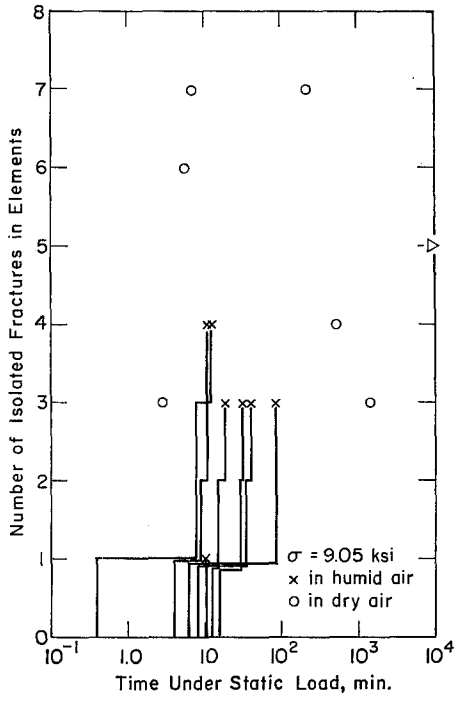

Figure 13 History of development of damage by continued fracture in elements in statically stressed laminates in humid air and dry air with a 3.0 in. test length.

From the above experiments, we conclude that static fatigue is at work in the slow fracturing of laminates either under a constant tensile stress or under a repeatedly applied tensile stress. Elimination of humidity from the air at constant stress, significantly increases the time to fracture in comparison with that for a laminate tested in laboratory air. Furthermore, since fracture is not completely eliminated in dry air, it appears that the humidity affecting the strength is, in part, entrapped on the interfaces between glass and polyethylene and continues to reduce the strength. Although this is surprising in view of the vacuum bonding technique used in making the laminates, it is well known that complete removal of the adsorbed water from glass surfaces requires baking the part out at a temperature considerably in excess of 225°C which was used in the vacuum bonding. In addition, it is well known that significant amounts of water are absorbed by many polymers in bulk and along polymer interfaces (for a review see McCrum [12]) making some re-contamination with water of the glass-polyethylene interfaces possible. The two outer glass surfaces of the laminate making up 20% of the total surface

area under stress must, of course, respond directly to the change in humidity and is, in all probability, responsible for the increase in life in the laminates tested in dry air.

4. Discussion

The experiments on the highly idealized glass ribbon-polyethylene laminates demonstrate clearly a number of important effects in fracture of composites which are not generally appreciated.

First, a "rule of mixtures" in which the average strength of a reinforcing element is taken and is combined with the average load carrying capacity of a matrix at the strain equal to the strain of fracture of the reinforcing elements, according to the volume fractions of the two components present, and ignoring the details of coupling between matrix and reinforcement is too crude to be of much value. As discussed by Argon [2] (see also Zweben [9]) the so-called "rule of mixtures" strength can be more or less than the actual composite strength depending upon whether the exponent m of the flaw distribution function in the reinforcement is greater than or less than a certain value (~ 10). In particular, in the experiments reported here, where $m = 4.2$ and a composite strength greater than the "rule of mixtures" strength is expected, we find that the actual average strength is 18.3 ksi, while the "rule of mixtures" strength based on the average strength of elements, is only 13.4 ksi as can be readily found from the mean value of the distribution 15.6 ksi given in Fig. 5 (for 3 in. test length) multiplied by the volume fraction of reinforcement, $V_r = 0.86$. The reason for this discrepancy, of course, is that the "rule of mixtures" strength ignores the statistical aspects of fracture in a laminate which result from the way in which neighbouring elements are mechanically coupled through the intervening matrix material.

It is of interest to compare our results with some other theories for tensile strength of composites, in which element coupling effects are considered, in particular that of Rosen [8] and the theory of Argon and Scop [6], both of which are *for large numbers of parallel elements* (of the order of 100 or more). The latter theory is based on a failure propagation model in which the effect of stress concentrations is considered. In the theory of Rosen [8], using our symbols, the strength of the composite is given by

$$\sigma_c = V_r\left[\frac{k}{Cwt_r\, em}\right]^{\frac{1}{1+m}} + (1 - V_r)\sqrt{3}\,k \qquad (4)$$

in which e is the Naperian number. The thickness t_r, of the sheets and k, the yield strength in shear of the matrix are introduced into Equation 4 through the unloading length, $\delta = \sigma_c t_r/k$, that gives the range of influence of an element fracture in the Rosen theory. The second term in Equation 4 represents the load carrying capacity of the matrix at yield. The Rosen composite strength computed from Equation 4 is 22.8 ksi. In the corresponding theory of Scop and Argon [6], the composite strength is given by (see Appendix 2)

$$\sigma_c = kV_r \frac{f(N, n, m)}{[(0.217)\, wt_r Ck^m (K_3{}^m - 1)]^{1/(2+m)}} + (1 - V_r)\sqrt{3}k \qquad (5)$$

where K_3 is the stress enhancement factor for three adjacent element fractures and is considered to be a good representation of the state of fracture instability. The factor $f(N, n, m)$, which is 0.858 in this case, is a weak function of the number N of parallel elements; n is the number of unloading lengths, δ, in series in a tensile laminate as described in Appendix 2. The stress enhancement factor K_3 which accounts for the increased probability of fracture in the entire overstressed region of the unfractured elements around a triple fracture has been calculated for a variety of exponents of flaw distribution functions by Argon [3]. The value of K_3 in this case, where $m = 4.2$, is 1.315. This gives for the composite strength in the Scop and Argon [6] theory, a value of 22.5 ksi. The reason for the over-estimates of these two theories is that they consider only a single sequential fracture mode. As has been shown by Scop and Argon [4], there are actually 2^{N-1} fracture modes in a laminate with N parallel elements. In laminates with small numbers of elements (such as five) where stress intensification is weak and is primarily a result of the reduction of the nominal load carrying area (see Neuber [13]) many of these fracture modes contribute to an equal extent to the probability of fracture across an area. Hence, a fracture model based on only one of these modes results in an under-estimate of the probability of fracture and an over-estimate of the composite strength. When the composite becomes very wide with many parallel elements, so that strong

stress intensification can develop around multiple fractures in adjacent elements, the sequential fracture propagation mode is dominant [6] and the estimates given by Equations 4 and 5 become quite adequate.

Secondly, the experiments show that under repeated tension, fatigue by propagation of delamination cracks starting from isolated stable fractures in reinforcing elements is a very slow process in a ductile matrix with a very high elastic strain at yield, as it is in the case of polyethylene. The process actually occurs but is overshadowed by continued further fracturing of reinforcing elements owing to a time-integrated stress corrosion cracking process. Although such a humidity-induced stress corrosion cracking process is well known in bulk glass and fibres (for a recent assessment of the problem see Wiederhorn [14] and McCrum [12]) and is probably widespread in fibre-glass composites [15], this latter fact does not seem to have been properly appreciated. We have shown here, both in single elements and even in manufactured laminates, that static fatigue plays a role in the fracture process and appears to be nearly entirely responsible for the damage in repeated application of a tensile stress. From this we conclude that elimination of moisture during lamination and sealing the laminate after manufacturing by a film impervious to humidity would be beneficial.

Furthermore, we have demonstrated that surface films and processes such as cleaning can have very marked effects on the strength of reinforcing elements through their stress corrosion cracking properties. Finally, we have shown possible evidence that lamination itself can produce degradation of strength, perhaps through surface scratching with dust particles and the like.

Appendix 1

Fracture of a five-element laminate

It was shown by Scop and Argon [4] that the fracture of a laminate of length L made up of N parallel elements can be viewed as fracture in the weakest link of a chain of n links where $n = L/\delta$ in which δ is the extent of unloading of a fractured element. The probability of fracture $G(s)$ of the entire laminate at a non-dimensional stress s or less is then given by

$$G(s) = 1 - [1 - F(s)]^n \qquad \text{(A-1)}$$

where

$$s = w\delta C\sigma_c{}^m$$

in which w = width of the elements, $\delta = \sigma_c t_r/k$, t_r = thickness of elements, k = yield strength in shear of matrix, C = coefficient of flaw density distribution, with dimensions stress^{-m} area^{-1}, σ_c = the tensile stress in the reinforcing elements, m = exponent of the flaw density distribution function.

The term $F(s)$, which represents the probability of fracture in a strip of length δ at a non-dimensional stress s or less, is made up of 2^{N-1} modes which may all be important. Let,

$$Q_i = 1 - \exp(-K_i{}^m s) \qquad \text{(A1-2)}$$

represent the probability of fracture in an element in a strip of length δ after i elements in the strip have already fractured. In Equation A1-2 the stress enhancement factor

$$K_i = \frac{N}{N-i} \qquad \text{(A1-3)}$$

takes account of only the nominal stress rise across the cross-section which is adequate when N is small (finite cross-section with large change in cross-sectional area, see Neuber [13]). In the five-element laminate, the probability of fracture, $F(s)$, in a strip of length δ at a stress s or less is then given by the following sixteen modes [4]:

$$\begin{aligned}
F(s) = {} & 120\, Q_0 (Q_1 - Q_0)(Q_2 - Q_1)(Q_3 - Q_2)(Q_4 - Q_3) \\
& + 60 [Q_0 (Q_1 - Q_0)(Q_2 - Q_1)(Q_3 - Q_2)^2 \\
& \quad + Q_0 (Q_1 - Q_0)(Q_2 - Q_1)^2 (Q_4 - Q_2) \\
& \quad + Q_0 (Q_1 - Q_0)^2 (Q_3 - Q_1)(Q_4 - Q_3) \\
& \quad + Q_0{}^2 (Q_2 - Q_0)(Q_3 - Q_2)(Q_4 - Q_3)] \\
& + 30 [Q_0 (Q_1 - Q_0)^2 (Q_3 - Q_1)^2 \\
& \quad + Q_0{}^2 (Q_2 - Q_0)(Q_3 - Q_2)^2 \\
& \quad + Q_0{}^2 (Q_2 - Q_0)^2 (Q_4 - Q_2)] \\
& + 20 [Q_0 (Q_1 - Q_0)(Q_2 - Q_1)^3 \\
& \quad + Q_0 (Q_1 - Q_0)^3 (Q_4 - Q_1)] \\
& + 12\, Q_0{}^3 (Q_3 - Q_0)(Q_4 - Q_3) \\
& + 10\, Q_0{}^2 (Q_2 - Q_0)^3 \\
& + 6\, Q_0{}^3 (Q_3 - Q_0)^2 \\
& + 5 [Q_0 (Q_1 - Q_0)^4 + Q_0{}^4 (Q_4 - Q_0)] \\
& + Q_0{}^5. \qquad \text{(A1-4)}
\end{aligned}$$

The evaluation of $F(s)$ and then $G(s)$ for various values of s is tedious and is best done by a digital computer. The value of s which makes $G(s) = 0.5$ is the average laminate strength for 100% reinforcement. The actual laminate

strength is then obtained by diluting this value in the ratio of V_r to unity, and by adding to it any contribution $(1 - V_r)\,\sigma_m$ derivable from the matrix, where σ_m is the stress in the matrix when fracture occurs. In the case under consideration, where $m = 4.2$, $C = 5.92 \times 10^{-6}$ in.$^{-2}$ ksi$^{-4.2}$, $t_r = 2.39 \times 10^{-3}$ in., $w = 0.463$ in., $L = 3$ in., $k = 2.12$ ksi, the resulting curve is shown in Fig. 6.

Appendix 2

Fracture of a laminate with a large number of parallel elements

Scop and Argon [6] showed that when the number, N, of parallel elements is large (of the order of 100 or more) and stress intensification is strong, a good estimate of the composite strength can be obtained by considering the dominant mode of sequential failure propagation alone. As amplified further by Argon [2], the resulting expression for the composite strength is

$$\sigma_c = V_r k \frac{f(N, n, m)}{[(0.217)\, wk^m t_r C\,(K_3^m - 1)]^{1/(2+m)}} + (1 - V_r)\sqrt{3k} \quad \text{(A2-1)}$$

where

$$f(N, n, m) = \left[\frac{0.5\,(K_3^m - 1)^3}{(K_1^m - 1)\, n\, N}\right]^{\frac{1}{N(2+m)}} \quad \text{(A2-2)}$$

is obtained from a truncated sequence of fracture propagation discussed by Scop and Argon [6], in which K_1 is the stress enhancement factor for a single fracture. In the derivation of Equation A2-1 the unloading range, δ, was taken as twice the plastic zone size d_p [2] i.e.

$$\delta = 2d_p = \frac{\pi^2}{16} r t_r \left(\frac{\sigma}{k}\right)^2 \sqrt{\left(\frac{a_{22}}{a_{66}}\right)} \quad \text{(A2-3)}$$

where r = number of fractured adjacent elements acting as a crack, 3 in this case, $a_{22} \simeq 1/V_r E_r$, average in-line compliance, $a_{66} \simeq (1 - V_r)/G_m$, average shear compliance. E_r = Young's modulus of reinforcement material and G_m = shear modulus of matrix.

The numerical constant (0.217) in Equation A2-1 is a result of the collection of factors in Equation A2-3, and the second term in Equation A2-1 represents the load carrying contribution of the matrix at yield.

Use of $E_r = 8.5 \times 10^3$ ksi, $G_m = 28$ ksi, $K_1 = 1.15$ [3], and $n = L/\delta$ where $L = 3$ in., together with the previously quoted other values gives the strength of the laminate as 22.5 ksi. As discussed earlier, this is an over-estimate, because the fracture probability was based only on the dominant mode out of a total of possible 2^{N-1} modes.

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